NAME:_____

GRADE:_____

SCHOOL NAME:_____

P	1	2	3	4	5	6	7	Total
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2025-2026 UAB MATH TALENT SEARCH

This is a two hour contest. **No electronic devices are allowed! Full credit is awarded only for a correct answer with correct justification. No credit is awarded if the answer is wrong. If the justification is incorrect, the grade is at the discretion of the graders.** Your work should be shown on the extra paper which will be provided. **THE ANSWERS MUST BE WRITTEN IN THE SPACE PROVIDED ON THE TEST!**

PROBLEM 1 (10 pts) The dwellers of an island belong to two tribes: the TRUTH-TELLERS and the LIARS. The TRUTH-TELLERS always tell the truth, the LIARS always lie. Nobody on the island knows the tribal status of anybody else. A person A sails to the island and asks a local person X to ask another local person Y whether he is a TRUTH-TELLER, and report to A what Y said. After talking to Y, X reported to A that Y claimed to be a TRUTH-TELLER. What is the tribal status of X?

YOUR ANSWER: X is a TRUTH-TELLER.

Solution: Y in any case well respond "Yes, I am a TRUTH-TELLER". Hence if X reported to A that Y is a TRUTH-TELLER then X is a TRUTH-TELLER.

PROBLEM 2 (30 pts) Jack and Jill went down the hill. Jack was walking; Jill was running with the speed seven times faster than Jack's. Exactly at the midpoint Jill fell down; even though she immediately got up, she was only able to walk afterwards with the speed which was half of Jack's. Who will be the first to reach the bottom of the hill?

YOUR ANSWER: Jack.

Solution: Since Jill walks with half the speed of Jack, he time she will spend reaching the bottom of the hill equals the overall time Jack needs to reach the bottom of the hill. However when she starts walking he will have already gone some distance towards the bottom of the hill. Hence he will be here first.

PROBLEM 3 (40 PS) In the convex quadrilateral ABCD the sides AB, BC, and CD are equal, the angle at C is 90 degrees, the angle at A is 45 degrees. Find the other two angles of the quadrilateral.

YOUR ANSWER: 75 degrees and 150 degrees.

Solution: Assume that $|AB| = |BC| = |CD| = 1$. Then $|BD| = \sqrt{2}$. By Law of Sines we have $\sqrt{2}/\sin(45^\circ) = 1/\sin(\angle ADB)$ which implies that $\angle ADB = 30^\circ$. Hence $\angle ABD = 105^\circ$, and so $\angle ADC = 75^\circ$ and $\angle ABC = 150^\circ$.

PROBLEM 4 (80 pts) Three diggers dig a hole. They take turns. Each works the same amount of time the other two need together to dig half the hole. After one round they complete digging the hole. How much faster will they dig the same hole if they work together?

YOUR ANSWER: 2.5 times faster.

Solution: Imagine that while one of the diggers digs the main hole, the other two dig extra holes. Then by the times they are done digging they will have dug $3 \cdot 0.5 = 1.5$ extra holes. Hence if they had worked together they would have dug 2.5 holes. This implies that together they dig 2.5 faster.

PROBLEM 5 (110 pts) There are five different integers on the board. They are the three coefficients and the two roots of the quadratic polynomial $ax^2 + bx + c$. Four numbers out of these five are 2, 3, 4, and -5. What is the missing fifth number?

YOUR ANSWER: -30.

Solution: Denote the roots of this polynomial by m and n. Then $c = amn, b = -a(m+n)$. Hence c is a multiple of three other numbers. Hence the missing number is c. Since b is a multiple of a then $a =$

$2, b = 4$, the numbers $3, -5$ are roots, and $c = amn = 2 \cdot 3 \cdot (–5) = –30$.

PROBLEM 6 (160 pts) Initially there are 7 empty boxes. Alice selects some of these boxes and puts 7 smaller separate boxes in each. Then she selects some of these new smaller boxes and puts 7 even smaller boxes in each. And so on. Alice stops when she has 10 non-empty boxes. How many boxes overall does she have in the end?

YOUR ANSWER: 77.

Solution: In the beginning we have 7 empty boxes. Each action fills one empty box. Since at the end we have 10 non-empty boxes there have been 10 actions. Each action adds 7 boxes, total of 70. In the beginning we had 7 boxes. Hence overall we will have 77 boxes.

PROBLEM 7 (250 pts) Bob and Bill play the following game. There are 31 coins in one pile on the table. The boys take turns. The first to go is Bob. The action is that he splits every pile of coins on the table that has more than one coin into two non-empty piles. The winner is the person after whose turn there are 31 piles of exactly one coin. Who wins and what strategy will allow him to win?

YOUR ANSWER: Bill.

Solution: Let us keep an eye on the size of the largest pile. In the beginning it is 31 $(= 2^n - 1)$. After Bob acts it will be between 16 and 30. Then Bill can make the largest pile size to be 15 $(= 2^{n-1} - 1)$ and the process continues. After several steps Bill will make the largest pile size to be 3. Clearly, on the next step Bob will reduce the largest pile size to 2, and then Bill wins.